[7] Robertson, J. M., "On Turbulent Plane Couette Flow," *Proceedings of the 6th Midwestern Conference on Fluid Mechanics*, University of Texas, 1959, pp. 169–182.

[8] Chue, S. H., "A Contribution to Boundary Layer Theory," Ph.D. thesis, School of Mechanical Engineering, Purdue University, Lafayette, Ind., June 1970.

[9] Chue, S. H., "An Experimental Investigation on Turbulent Plane Couette Flow in Dilute Polymer Solutions," M.S. thesis, School of Mechanical Engineering, Purdue University, Lafayette, Ind., Jan. 1969.

[10] Clauser, F. H., "Turbulent Boundary Layers in Adverse Pressure Gradients," *Journal of the Aeronautical Sciences*, Vol. 21, No. 2, Feb. 1954, pp. 91–108.

Rotational Temperature Measurements in Low-Density Flows

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Introduction

THE electron beam fluorescence technique has been described by Muntz.[1] A narrow beam of electrons is passed through the gas and from the intensity of fluorescence the density can be determined, whereas for nitrogen the rotational temperature can also be obtained from the relative intensities of rotational lines in the spectrum. Muntz[1] proposed a mathematical model which relates the measured line intensities of the first negative system of nitrogen to the population of the rotational levels in the unexcited gas. If this population distribution is Maxwellian, then the rotational temperature of the gas can be calculated.

Experimental studies have shown that although density measurements are reliable,[2] the rotational temperature calculated using Muntz's model is higher than the true temperature.[3] In an earlier Note,[4] this discrepancy was tentatively explained in terms of excitation by secondary electrons, although there is still some doubt about the exact process involved. Using Ashkenas'[3] experimental results, Muntz's theory was modified to include the assumed secondary electron excitation leading to the expression for the relative line intensities

$$I(K')/(2K'[G_p + n \cdot G_s]) \; \alpha \; [e^{-\phi K'(K'+1)/T_R} \qquad (1)$$

where $I(K')$ is the relative line intensity and K' is the rotational quantum number. G_p is a function of K' and T_R given by Muntz[1] for the primary electrons and G_s is a similar but unknown function for the secondary electrons. The number density is denoted by n, whereas $\phi = 2.88$ for the ground state vibrational level.

These electron beam fluorescence techniques have been used to study rarefied flow over a sharp leading edge flat plate. From the measured density and the rotational temperature calculated by using Eq. (1), the static pressure in the shock layer was obtained. The variation through the shock layer for the weak interaction and merged regimes was examined and the pressure in the viscous layer compared with the measured surface pressure corrected for orifice effects.

Experimental Details

The experiment was performed in the NPL (National Physics Laboratory) Low-Density Tunnel using pure nitrogen at a stagnation temperature of 632°K. The freestream Mach number, calculated from the ratio of the stagnation pressure to freestream pitot pressure corrected for viscous effects, was 6.28 and the Reynolds number was 370 per cm. A water-cooled model was mounted in front of the electron gun and joined to it by a narrow drift tube. The electrons pass along this tube and emerge through a 0.5-mm-diam hole in the model, normal to the surface. More details are given in Refs. 2 and 5. In all tests the pressure in the drift tube was maintained equal to that on the surface of the model to eliminate any flow of gas through the tube.

An optical system produced an image of the beam at the entrance slit of the spectrometer orientated so that the length of the beam was perpendicular to the slit. The spectrometer could resolve all the lines in the 0–0 band of the first negative system of nitrogen sufficiently for the peak intensity to be proportional to the line intensity. A portion of the light beam incident on the entrance slit was deflected by a small mirror onto a photomultiplier so that the output was proportional to the total intensity of fluorescence and served as a monitor for variations in flow conditions or beam current while measuring the line intensities of the 0–0 band.

Density and rotational temperature profiles through the shock layer were obtained at several positions along the centre line of the model. Density profiles were obtained in the usual manner[2] by measuring the beam current and the intensity of fluorescence at several points through the shock layer. At each point the relative intensity of the first 11 lines of the 0–0 band were also measured by rotating the grating.

Analysis of Rotational Spectra

If Eq. (1) is correct a plot of $\log_e$ of the left hand side against $K'(K'+1)$ will be a straight line, the slope of which is ϕ/T_R. A problem arises when applying this equation since G_s is a function of T_R and in Ref. 4 values of G_s/G_p could only be obtained at 78°K and 289°K. However, for the first 11 lines G_s/G_p does not vary strongly with T_R over this range, and therefore, can be assumed to vary linearly without making serious errors.

Except when near the surface of the model, the measured intensities gave a good linear plot although in some cases there was a small systematic increase in temperature as the number of lines used to calculate the temperature was reduced, possibly due to inaccuracies in G_s. The gradients in the flow regime investigated do not appear to be large enough to cause any nonequilibrium between rotational and translational degrees of freedom.

Within 2 or 3 static mean free paths of the surface the plots become very nonlinear. Assuming that the rotational and translational degrees of freedom are in equilibrium, this indicates that the velocity distribution function is non-Maxwellian, as might be expected if there is an appreciable temperature jump at the surface. Measurements were made to within half a mean free path of the surface and attempts were made to analyze the rotational spectra in terms of a two-stream Maxwellian model. In all cases the rotational temperature computed for the incident stream increased with the number of lines used in the calculation to such an extent that the values could not be regarded as reliable.

Rotational temperature profiles obtained using all eleven lines are shown in Fig. 1. If on increasing the number of lines used to calculate T_R from 7–11 there was a systematic increase in the temperature of more than 10%, the data was

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rejected. For most cases the systematic increase in T_R was less than 4%, but was more pronounced close to the surface. Values for the freestream rotational temperature agree with the calculated static temperature assuming an isentropic expansion through the nozzle to within $^{+2}_{-0}\%$.

Surface Pressure

The surface pressure was measured through 0.5-mm holes in the model surface and must be corrected for the orifice effect. Harbour and Bienkowski[6] point out that using techniques such as that of Kinslow and Arney[7] and Bartz and Vidal[8] could lead to errors as the leading edge is approached, due to the large shear stress at the wall. They[6] proposed two alternative correction schemes, which near the leading edge lead to a downward correction in the measured surface pressure in contrast to the other methods.[7,8] However, all the correction schemes should result in an upward or downward correction depending on whether the temperature of the molecules approaching the wall is greater or less than that of the molecules reflected from it. The discrepancy arises from using the measured heat transfer in 7 and 8 under conditions where the slip velocity at the surface cannot be neglected. Bartz and Vidal used slip boundary conditions consistent with the Navier-Stokes equations so that the heat transfer to the surface is given by

$$q_w = -(k\partial T/\partial Y + \mu u \partial u/\partial Y)_G \qquad (2)$$

where k is the thermal conductivity, μ is the viscosity and G denotes conditions in the gas. The temperature measurements, Fig. 1, and estimates of T_{Gw} in Ref. 5 show that close to the leading edge the convection term $(k\partial T/\partial Y)_G$ could be negative or positive depending on T_w/T_0. Under these conditions the importance of the slip velocity in determining the energy transfer to the surface must be considered.

Bartz and Vidal assumed the effect of the velocity terms in their expression would be small and therefore neglected them. The upper curve in Fig. 2 shows the present data corrected by their method with velocity terms neglected. Hendry's[9] heat-transfer measurements for $M = 5.51$ and 5.59 were extended to $M = 6.28$ by a correlation of $M^3C_H/\chi^{3/2}$ against $M^2(C_\infty/Re_{x,\infty})^{1/2}$ and used to make these corrections. C_H is the Stanton number given by $q/\rho_\infty U_\infty(H_\infty - h_w)$. To estimate the effect of the velocity terms it was assumed that the skin-friction coefficient, $C_f \simeq 2C_H$ since Vidal and Bartz[10] have shown that the Reynolds analogy is approximately true. Including the velocity terms in this way leads to much smaller corrections towards the leading

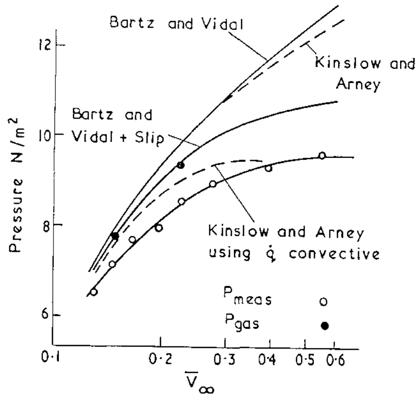

Fig. 2 Surface pressure.

edge as shown by the curve marked Bartz and Vidal + slip.

Kinslow and Arney considered the energy transfer to the surface because of the difference in temperature between the incident and reflected molecules which for convenience is called the convective heat transfer. If this process is assumed to be undisturbed by the tangential velocity of the gas, an estimate can be made of the effect of slip on the orifice correction. The slip-boundary conditions have been used to calculate $(\mu u \partial u/\partial y)_G$ in Eq. (2) and hence obtain an estimate of the convective heat transfer. This will be inaccurate for it is obtained from the difference of two similar terms, but it can be used to illustrate the effect of slip as in Fig. 2. When the measured heat transfer was used, the corrected pressures were much higher than those obtained using the estimated convective heat transfer only.

Figure 2 clearly shows that for $\bar{V}_\infty < 0.15$ the two correction schemes are in good agreement and the effects of slip are small. It was important to establish this since Harbour and Bienkowski's method could not be used because the pressures were not measured through free molecular orifices. For $\bar{V}_\infty > 0.15$ reliable orifice corrections can only be applied if accurate measurements of heat transfer and skin friction are made.

Discussion of Results

The static pressure variation through the shock layer has been calculated using the equation of state, Fig. 3. The profile 4.45 cm from the leading edge is in the weak interaction regime[2] and confirms the reliability of the rotational

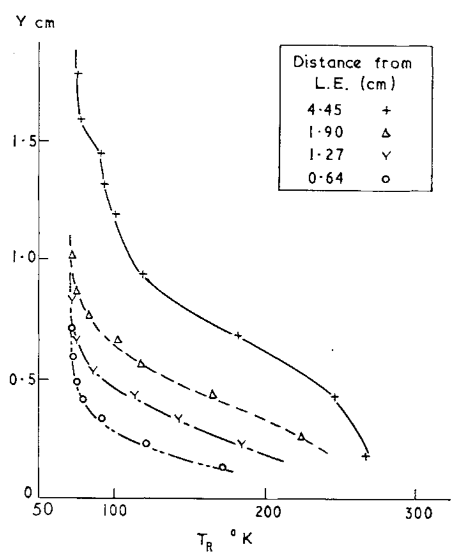

Fig. 1 Rotational temperature profiles through the shock layer: $M = 6.28$, $T_w/T_o = 0.46$, $Re_\infty/\text{cm} = 370$.

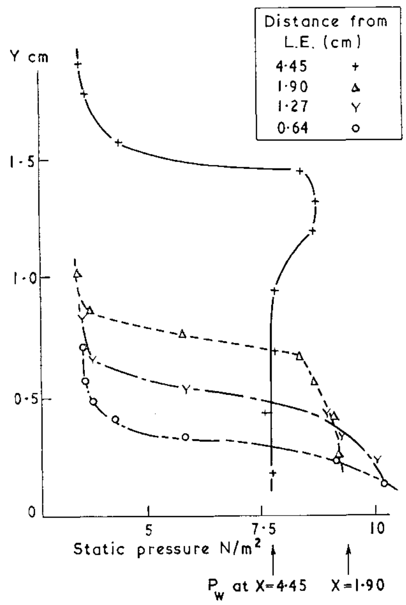

Fig. 3 Static pressure variation across shock layer $M \simeq 6.28$, $T_w/T_o = 0.46$, $Re_\infty/\text{cm} = 370$.

temperature measurements. As required for a weak interaction regime an inviscid region separates the shock wave from a constant pressure viscous layer where the pressure is equal to the surface pressure. This viscous layer pressure is also shown in Fig. 2 and is in very good agreement with the corrected surface pressure when an estimate is made for the effect of slip. In addition, the static pressure and temperature that rise across the shock wave agree with the Rankine-Hugoniot values to within 4%. Also, the freestream rotational temperatures agreed with the calculated static temperature to within $^{+2}_{-0}\%$. Although there is still some doubt about the secondary excitation process involved, these measurements demonstrate that Eq. (1), based on the experiments of Ashkenas, does not result in serious systematic errors over the temperature range 70°K to room temperature, at least for the density levels encountered in this experiment.

The profiles for the merged regime show that the static pressure increases across the viscous layer, the gradients becoming larger as the leading edge is approached. In this regime the shock wave is not of primary importance in determining the surface pressure, and the static pressure behind the shock will only equal the surface pressure at the point where merging occurs. Because of the increasing uncertainty in T_R as the surface is approached, it is dangerous to extrapolate these profiles to the surface, although the pressure profile 1.9 cm from the leading edge shows reasonable agreement with the curve of Bartz and Vidal + slip.

There are no known theoretical results with which these measurements can be compared directly. Oguchi[11] estimated that for a highly cooled wall the normal pressure gradient was small and this has been used in many theoretical models. Huang and Hwang[12] have computed profiles for $M = 5$ and $T_w/T_0 = 0.15$ which show considerable variations in static pressure across the viscous layer. The present results show that a constant pressure viscous layer model cannot be used for higher wall temperatures, and there is a need for further studies of this type to check Oguchi's estimate of a small normal pressure gradient for a highly cooled wall.

References

1 Muntz, E. P., "Measurement of Rotational Temperature, Vibrational Temperature and Molecular Concentration of Low Density Nitrogen," *The Physics of Fluids*, Vol. 5, No. 1, 1962, pp. 80–90.

2 Lillicrap, D. C. and Berry, C. J., "An Experimental Model for High Speed Rarefied Flow Over a Sharp Flat Plate," *The Physics of Fluids*, Vol. 13, No. 5, 1970, pp. 1146–1152.

3 Ashkenas, H., "Rotational Temperature Measurements in Electron Beam Excited Nitrogen," *The Physics of Fluids*, Vol. 10, No. 12, 1967, pp. 2509–2520.

4 Lillicrap, D. C. and Harvey, J. K., "Electron Beam Rotational Temperature Measurements Including the Effect of Secondary Electrons," *AIAA Journal* Vol. 7, No. 5, May 1969, pp. 980–982.

5 Metcalf, S. C., Lillicrap, D. C., and Berry, C. J., "A Study of the Effect of Surface Temperature on the Shock-Layer Development over Sharp-Edged Shapes in Low-Reynolds-Number High Speed Flow," *Rarefied Gas Dynamics*, edited by Trilling and Wachman, Vol. 1, Academic Press, N. Y. 1969, pp. 619–634.

6 Harbour, P. and Bienkowski, G. K., private communication, June 1969, Imperial College, London.

7 Kinslow, M. and Arney, G. D., "Thermo-Molecular Pressure Effects in Tubes and at Orifices," AGARDograph 119, 1967.

8 Bartz, J. A. and Vidal, R. J., "Experimental Study of Pseudo Transpiration at an Orifice in Rarefied Flow." AFOSR 68-1927, 1968, Cornell Aeronautical Lab. Inc., Vandenburg, Calif.

9 Hendry, J. D.: "Local Leading Edge Heat Transfer on a Flat Plate in Rarefied Hypersonic Flow," Rept. AS-63-8, 1963, Univ of California, Berkeley.

10 Vidal, R. J. and Bartz, J. A., "Surface Measurements on Sharp Flat Plates and Wedges in Low-Density Hypersonic Flow," A.F.O.S.R. 68-0645, 1968, Cornell Aeronautical Lab., Inc., Vandenburg, Calif.

11 Oguchi, H., "Shock Wave and Viscous Layer Structure in a Rarefied Hypersonic Flow Near the Leading Edge of a Sharp Flat Plate" Rept. 418, 1967, Univ. of Tokyo, Tokyo, Japan.

12 Huang, A. B. and Hwang, P. F., "Kinetic Theory of the Sharp Leading Edge Problem II Hypersonic Flow," *Proceedings of the 19th Congress of the International Astronautical Federation*, N. Y., 1968.

Application of Whitham's Theory to Sonic Boom in the Mid- or Near-Field

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Introduction

In experimental investigations of sonic boom problems in wind tunnels, it is usually necessary to use very small models in order to get direct results for the far-field. With these methods, inaccuracies due to the model contours, boundary-layer development, etc., usually arise. A new method is based on large models in wind tunnels where only the near- or mid-field is simulated. The pressure signatures of the models in the far-field are calculated from the pressure distributions measured in the vicinity of the wind tunnel wall. The present investigation is directly related to this new method. That is, to extrapolate a known pressure signature in the near- or mid-field to the far-field.

Calculations of sonic boom pressure signatures of an aircraft have been based mainly on Whitham's theory,[1] which describes the asymptotic behavior of the flowfield far from an equivalent body of revolution of the aircraft. Investigators on sonic boom, however, have used this theory to predict nonasymptotic pressure signatures in a nearer field.[2,3] In the mid- or near-field, Whitham,[4] Lighthill,[5] and more recently, Moore and Henderson[2] have considered asymptotic expansions of Whitham's theory in order to obtain certain modifications. However, to the author's knowledge, no general quantitative modifications of Whitham's theory in the mid- or near-field have been obtained.

In this Note, we shall be concerned with the validity of Whitham's theory in the mid- or near-field of a slender body. We shall be particularly interested in the quantitative and qualitative modification of Whitham's theory and in the extrapolation of a known disturbance signature to the far field.

Asymptotic Relations of Whitham

For simplicity, we consider a steady, homogeneous, inviscid, supersonic flow over a slender body of revolution. It is clear that the disturbance of the flowfield due to the presence of a slender body is equivalent to that caused by a streamtube (i.e., a quasi-cylinder) enclosing this body in a nearer field. Therefore, instead of considering the flowfield of the body, we shall consider the flowfield over a streamtube enclosing this body.

Let us choose the axis of the body to be the x axis coinciding with the freestream direction (Fig. 1). Enclosing this body, we choose a coaxial circular streamtube with a radius R. On the streamtube surface, the flow starts to be disturbed at $x = 0$, where the origin of the x axis is defined. The r axis perpendicular to the x axis is the radial coordinate.

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